Improved Procedures for Determining Surface Pressures in Low-Density Hypersonic Flow

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Depending on the type of surface pressure measuring technique used, there are often difficulties associated with accurately determining surface pressures on bodies in low-density hypersonic flows. The errors arising in the three most commonly used measuring techniques are discussed, and a means of converting the measured pressure to the surface pressure is given for each technique. The inadequacies of previous conversion or correction schemes are assessed through comparisons of experimental data. Guidelines are deduced by using two of the most popular techniques to measure pressures on a flat plate model placed in a hypersonic flow where $M_\infty = 25$, $Re/x \sim 1000$/in.

Introduction

MANY investigations of the flow of a hypersonic low-density gas about a body have produced measurements of surface pressures. However, the accuracy and hence the interpretation of these data is dubious in most cases due to the large corrections, typically 10%–200% of measured values, which have been applied to the raw data. The major corrections arise from the effects of low density and hypersonic velocities near the surface. The low densities produce noncontinuum flow in the pressure orifices and tubes connecting the orifices to the transducers, thereby permitting a gradient in normal momentum flux in the connecting tubes even though there is no net molecular flux. The hypersonic velocities near the surface make the geometry of the entrance to the pressure orifice much more important particularly when the surface is aligned with the freestream. In the latter case the normal momentum transfer (surface pressure) depends only on the thermal velocity which is much smaller than the directed velocity, and therefore, small changes in orifice inclination can lead to large differences in measured pressure.

Various correction schemes including or excluding various phenomena have been proposed.[1–4] However, one cannot use the previously reported measured surface pressures to infer which of these correction schemes is applicable either due to lack of information about the experimental situation or, more disturbing, contradiction between measurements made in a similar flow. For example, compare the reported difference in the corrected wall pressure data presented in Refs. 5 and 6. These data were taken in the same wind tunnel, with the same nominal freestream conditions, using the same correction scheme,[2] and there exists a discrepancy of from 20% to 70% in the data nearest the leading edge. These data were obtained using two different wind-tunnel models. Additional unpublished measurements on flat plates made in this laboratory and on cones made elsewhere[7] testify to the lack of reproducibility from model to model. From the work presented here, we conclude that this lack of reproducibility arises from the extreme sensitivity to angle of inclination and length to diameter ratio which is exhibited by tube-orifice pressure ports in a low-density hypersonic flow condition. One other disturbing fact is that no one has reported surface pressures which approach the appropriate free molecule limit near the leading edge. In contrast, values of heat-transfer shear stress[2] and molecular flux[8] have been measured on similar models in similar flows and have been shown to approach their appropriate free molecular limit near the leading edge.

After describing the general features of the three measuring techniques presently available, we will concentrate on the measurements of surface pressures on sharp leading-edge flat plate models at zero angle of incidence with respect to a hypersonic freestream. We will report new measurements of surface pressure which approach free molecular values near the leading edge and extend to values which correlate with hypersonic strong interaction theory downstream. The surface pressures reported here were obtained by using measured values of surface molecular flux and number density in conjunction with a kinetic theory model for the flow. Under rarefied flow conditions, these results permit a direct evaluation of the various pressure correction schemes and provide a basis for general guidelines for measuring surface pressures on bodies with inclined and/or curved surfaces in hypersonic low-density flows.

Present Measuring Techniques

There exist three basic techniques for measuring surface pressures in low-density, hypersonic flows. They will be referred to as the flush transducer technique, the tube-orifice technique, and the orifice-cavity technique. The important differences in instrumentation geometry are shown in Fig. 1. These differences and the relative merits of each of these techniques will be discussed in detail.

Flush Transducer Technique

As indicated in Fig. 1a, the flush transducer technique consists of using a pressure transducer mounted flush with the surface of the model. This is the best approach since there are no low-density effects, per se, associated with this technique. The major disadvantages are associated with the development of the instrumentation. It is difficult to construct these devices so that they are isolated from the effects of tangential or shear stresses since the most common type of transducer is a piezoelectric crystal which may be sensitive to stress in more than one direction. Also, in order to preserve good spatial resolution, the sensing elements must be made very small without a corresponding loss in signal level. This last factor has precluded their use in all but the largest models used in the largest low-density hypersonic wind tunnels.[9,10] Another development difficulty arises when one considers using these devices on models with curved surfaces, since the sensing element size should be much smaller than the local surface radius of curvature.

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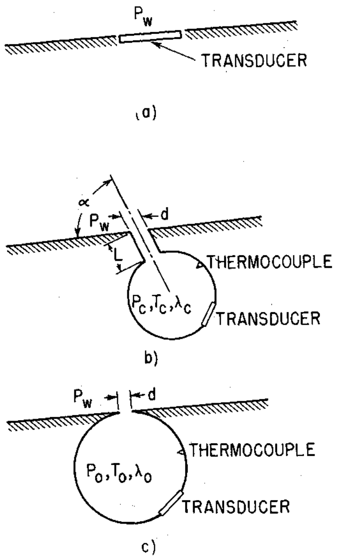

Fig. 1 Schematic diagrams of measuring techniques.

Tube-Orifice Technique

The tube-orifice technique consists of a small tube mounted flush with the surface of the model on one end and connected to a relatively large cavity containing a pressure measuring device on the other end (Fig. 1b). (In practice, there is often a length of relatively large diameter tubing acting as the cavity between the entrance orifice and the transducer.) This is by far the most often used technique, and it has the advantage of providing very good spatial resolution with relatively low fabrication costs. However, as has been long recognized, its usefulness in low-density hypersonic flows is, in many cases, limited. When the density is so low that the mean free path in the tube and cavity λ_c becomes larger than the tube diameter d, the pressure measured in the cavity P_c can be substantially different from the wall pressure P_w due to differences in temperature between the gas next to the surface and the gas in the tube and cavity, the so-called thermomolecular pressure effect described by Knudsen.[11] In addition, in a flowing gas some of the molecules entering the tube may strike the walls of the tube and re-enter the stream without equilibrating in the large cavity, and these molecules will not contribute to the measured value of P_c, though they would contribute to P_w were the orifice absent.

Two empirical correction schemes have evolved for determining P_w from measured values of P_c.[1,2] They have some features in common. Both require measurements of heat transfer to the surface at the location of the tube orifice. Both corrections depend upon the tube-orifice Knudsen number λ_c/d. Both assumed the measured cavity pressure P_c is equivalent to an orifice pressure P_0, where P_0 is defined as the cavity pressure measured when $L/d \to 0$. This assumption was justified by the failure to observe any differences in P_c as L/d was varied from 0.3 to 7 in one case[2] and from 0.5 to 3 in the other.[1] This assumption was required in each theoretical formulation because it was implicitly assumed that every molecule entering the entrance plane of the tube became trapped and thus contributed to P_c. If this were true, then P_c would indeed equal P_0.

While this assumption was clearly stated in Refs. 1 and 2, all the implied restrictions on the empirical correction schemes were not fully appreciated by those applying these corrections to tube measured pressures. These restrictions were made more evident in a well written paper by Hughes and DeLeeuw[12] in which computed values of P_c/P_o were given for a wide range of conditions. They used a free molecule theory to compute the percentage of entering molecules which struck the tube walls and were reflected back out, thus failing to contribute to P_c. They considered various combinations of incident gas speed ratio S_i length to diameter ratio of the tube, L/d, and tube inclination to the mean flow, α. The indicated departure of α from 90° in Fig. 1b is greatly exaggerated. Since the Hughes and DeLeeuw theory considered a tube whose entrance plane was perpendicular to the tube axis, the departure from $\alpha = 90°$ must be small. The validity of their theory will be established later. Their results showed not only the more obvious conclusion that for small S_i the effects of L/d are small, depending on α, but they also showed that if L/d was 0.3 or larger, the ratio P_c/P_0 was very insensitive to L/d for a given S_i and α, whereas a very large dependence on L/d was shown for $0 < L/d < 0.3$. They show, however, that P_c/P_0 was a very strong function of S_i and α for $L/d > 0.3$. This dependence on S_i and α was not revealed in the empirical correction schemes.

In developing their empirical correction scheme, Potter et al.[2] made measurements for $S_i \equiv 0$, and thus, they could not and did not observe any dependence on L/d or α. The failure of Vidal and Bartz,[1] on the other hand, to observe any effect of L/d is more subtle. They used conventional velocity slip and temperature jump estimates within the Navier-Stokes formalism. This gave an implicit equation for P_w in terms of P_0, T_0, $\dot{q}_w$, τ_w, and an empirical factor K which depends on the Knudsen number λ_c/d. Using a flat plate model immersed in a $M_\infty \simeq 20$, low-density stream inclined to the flow at angles from 0° to 40° (compression), they measured P_0, T_0, $\dot{q}_w$, and τ_w, and computed P_w using the implicit equation. These results were compared to measured values of P_w obtained by the flush transducer technique, and the difference varied from 0 to 40%. One might argue about the validity of the slip assumption under rarefied conditions, especially where shear forces are large. More serious is the fact that while it was assumed in the theory that P_0 corresponded to the pressure measured in a zero L/d, sharp-edged orifice cavity (Fig. 1c), the measurements of P_0 used were obtained from a tube-orifice with L/d's of 0.5 and 3. In effect they measured P_c and not P_0. The fact that P_c/P_0 varies with S_i and α for L/d fixed (or large)[12] probably accounts for their differences between the calculated and measured values of P_w.

The conclusion is then that these empirical correction schemes[1,2] employing measured values of $\dot{q}_w$, P_c, and T_c are useful in determining P_w only when S_i is small because the effects of L/d and α are diminished. Of course, in many cases S_i is not small, and one must then use the theory of Hughes and DeLeeuw[12] to find P_0 provided that $\lambda_c > d$ and one measures S_i, L/d, and α. (T_0 is assumed to be equal to the model wall temperature T_w.) In the next section we will describe an approach that can be used to calculate the wall pressure P_w from measured values of P_0, T_0, and n_m. n_m is the measured molecular number density in the entrance plane of a sharp edged orifice, i.e., at the model surface.

Orifice-Cavity Technique

As can be seen in Fig. 1c, the orifice-cavity technique is essentially a special case of the tube-orifice technique, i.e., the tube has zero length. The sharp-edged orifice in the model surface must have a diameter d which is smaller than a mean free path of the gas in the cavity λ_0, i.e., $d/\lambda_0 \ll 1$. The dimensions of the cavity, on the other hand, are of the order of λ_0 or larger. Under these conditions there exists no ambiguity in measured values of P_0 arising from entrance orifice geometry. The wall pressure P_w can be calculated from measured values of P_0, T_0 and n_m if we use a two stream Maxwellian model to describe the molecules entering and leaving the orifice. (The two stream Maxwellian distribution was originally introduced by Lees[13] for gasdynamic applications.) In essence we will assume that the molecular flux through the free molecular orifice consists of a superposition of an incident stream having a directed velocity and described by the half Maxwellian distribution function

$$f_i(\xi_x, \xi_y, \xi_z) = \frac{n_i m^{3/2}}{(2\pi k T_i)^{3/2}} \exp - \left\{ \frac{m[(\xi_x - U_i)^2 + (\xi_y - V_i)^2 + \xi_z^2]}{2kT_i} \right\}$$
$$\xi_y > 0$$

and a reflected stream having no directed component of velocity described by the half Maxwellian distribution function

$$f_r(\xi_x, \xi_y, \xi_z) = \frac{n_r m^{3/2}}{(2\pi k T_r)^{3/2}} \exp - \left\{ \frac{m(\xi_x^2 + \xi_y^2 + \xi_z^2)}{2kT_r} \right\}$$
$$\xi_y < 0$$

The x direction is aligned with the tangential component of the directed velocity of the incident molecules, and the y direction is the inward normal to the surface, and U_i and V_i are the corresponding components of the directed velocity. In the subsequent analysis, which corresponds to the conditions in which measurements were made, we will assume the following. 1) The temperature of the reflected particles T_r is equal to the wall (orifice-cavity) temperature T_0 which implies that there is complete translational accommodation. This assumption has been verified by correlations of heat transfer and shear stress measurements when complete accommodation was assumed[9] and by comparison of density profiles obtained near cavity-like diffusely reflecting surface areas and those obtained near ordinary aerodynamic test surfaces.[14] 2) The incident angle of the molecules is small implying that $V_i/C_i \ll 1$ which simplifies the analysis. C is used to denote the most probable thermal speed $(2kT/m)^{1/2}$.

The following comments should be made concerning these assumptions before proceeding. 1) The assumption that the emitted (reflected) stream has no directed component of velocity implies that molecules colliding with the surface are diffusely reflected. This assumption is supported by results from low-energy molecular beam scattering experiments on "engineering" surfaces.[15,16] 2) If a temperature difference exists between the cavity and the pressure transducer, the measured pressure P_a may have to be corrected for thermal transpiration, i.e., $P_0 = P_a(T_a/T_0)^{1/2}$ or an equivalent empirical correction if $\lambda_0 \sim$ cavity dimension.[4] Therefore, when the transducer is located some distance from the cavity it is imperative to measure both temperatures T_0 and T_a. 3) In cases where V_i/C_i is not very much less than unity, another quantity such as speed ratio S_i or heat transfer rate $\dot{q}_w$ will be required. Fortunately, in many rarefied hypersonic flows this assumption is valid, and when it is not, the pressure (density) levels are sometimes high enough to permit use of the tube-orifice technique.

Using the two stream Maxwellian model one can write the appropriate moments of the distribution function to determine such quantities as incident molecular flux, normal and transverse momentum exchange, etc., in terms of the incident velocity components U_i and V_i and the appropriate temperatures of the two streams T_i and T_r.[17] For the special case when $V_i/C_i \ll 1$, which is the one of primary interest for the experimental results to be presented here, these moments are particularly simple expressions which give a relationship between P_w and the measured quantities P_0, T_0, and n_m.[18]

Molecular flux per unit area

$$N_i = n_i C_i/2\pi^{1/2}; \quad N_r = n_r C_r/2\pi^{1/2}; \quad N_0 = P_0/(2\pi m k T_0)^{1/2} \tag{1}$$

Continuity at the gas-solid boundary and across the entrance plane of the orifice requires that

$$N_i = N_r = N_0 \tag{2}$$

Thus measurement of P_0 and T_0 determine surface molecular flux rather than the surface pressure P_w. However, we can write an expression for the normal momentum exchanged at the surface (wall pressure)

$$P_w = P_i + P_r = m n_i C_i^2/4 + m n_r C_r^2/4 \tag{3}$$

Using Eqs. (1) and (2) in (3) we can write another expression for P_w which involves the measured quantities P_0 and T_0

$$P_w = P_0^2/(4kT_0 n_m - 2P_0) + P_0/2 \tag{4}$$

where

$$n_m = \tfrac{1}{2}(n_i + n_r) = \tfrac{1}{2}(n_i + n_0), \quad P_0 = n_0 k T_0 \tag{5}$$

Thus, by measuring P_0, T_0, and n_m, we can determine P_w from Eq. (4). We can also write the following expressions.

Tangential momentum exchanged at the surface

$$\tau_{yx} = m n_i C_i U_i/2\pi^{1/2} \quad \text{or} \quad C_f = \tau_{yx}/\tfrac{1}{2} n_\infty m U_\infty^2 \tag{6}$$

Net energy flux to the surface

$$\begin{aligned} \dot{q}_w &= (m n_i C_i/2\pi^{1/2})\{(C_i^2 - C_r^2) + U_i^2/2 + (k/m)(T_{R,i} - T_{R,r})\} \\ C_H &= \dot{q}_w/m n_\infty U_\infty (H_\infty - h_w) \end{aligned} \tag{7}$$

Consistent with our previous restrictions the rotational temperature of the incident particles can be calculated from the measured rotational temperature in the entrance plane of the orifice $(T_{R,m})$ by summing the total rotational energy from the two streams

$$\begin{aligned} n_m T_{R,m} &= \tfrac{1}{2}(n_i T_{R,i} + n_r T_{R,r}) \\ &= \tfrac{1}{2}(n_i T_{R,i} + n_0 T_{R,0}) \end{aligned} \tag{8}$$

With this model of the gas-surface interaction one can, instead of measuring P_0, T_0, and n_m to determine P_w, use measured values of P_0, T_0, $\dot{q}_w$ and an assumed ratio of C_H/C_F. In principle, any combination of the moments of the distribution function can be used for this purpose.

In the next section we will discuss an experimental investigation in which the tube-orifice and orifice-cavity techniques were applied under identical conditions. From these results we can establish the validity of the Hughes and DeLeeuw theory[12] and can demonstrate the inadequacy of the empirical correction schemes used to convert tube-orifice pressure to wall pressures. However, to use the theory of Hughes and DeLeeuw in converting tube-orifice measured pressures to equivalent orifice-cavity pressures we need to know the value of $S_i = U_i/C_i$. C_i can be determined from Eqs. (1, 2, and 5) using measured values of P_0, T_0, and n_m. U_i in this investigation was determined in two ways: 1) $\dot{q}_w$ and $T_{R,m}$ were measured and U_i calculated using Eqs. (7) and (8), and 2) Vidal and Bartz's[1] measured ratio of C_H/C_F was assumed, and this coupled with the measured rotational temperatures $T_{R,m}$ was used to calculate U_i from Eqs. (6–8). Both methods gave the same values for U_i. (For a monatomic gas one needs only measure $\dot{q}_w$ to determine U_i.)

Experimental Investigation

Tests were conducted in the Princeton University low-density hot nitrogen hypersonic wind tunnel operating at $M_\infty \sim 25$ and $Re_{x\infty} \sim 1000$–1700/in. Various surface properties were measured on three similar sharp leading-edge nickel flat plate models aligned with the flow. The models were all $\frac{1}{8}$ in. thick by 3 in. wide by 6 in. long with a 10° bevel on the underside at the leading edge. The test surfaces were ground flat with a ± 5 μin. finish, and the wall temperature was kept constant (300°K) by circulating water through cooling tubes on the underside.

Since the details of the measuring procedure are described in detail elsewhere[18,8] only the briefest description of the experimental procedure appears here. One model containing nine orifice-cavities located from 0.25 in. to 4.0 in. from the leading edge was used to obtain values of P_0 and T_0 (= 300°K). The sharp-edge entrance to the orifice cavities had a diameter $d = 0.018$ in. ($\lambda_0/d \sim 25$) with a 12° bevel on the inside, and the cavities were connected to pressure transducers by 0.250 in. i.d. stainless-steel tubing. This model was also equipped with a 0.032-in.-diam tube-orifice located 1.0 in. from the leading edge. The pressure measured in this tube-orifice will be compared to orifice-cavity measurements at the same location.

Another model was connected on the under side to an instrumented elctron beam drift tube. An electron beam passed through the drift tube and then through a sharp edge orifice ($d = 0.036$ in.) in the model surface. The number density n_m and rotational temperature $T_{R,m}$ at the surface were measured by observing the total intensity and the rotational fine structure of the 0-0 vibrational band of the first negative system of N_2. The drift tube pressure and temperature were monitored, and small amounts of nitrogen could be bled into the drift tube to ensure that no net molecular flux through the model surface was produced by the electron gun pumping system.[18]

A third model containing three small parallelepiped glass elements with vacuum deposited nickel films on one surface was used to measure the heat transfer rate to the surface $\dot{q}_w$. The thin film resistance (temperature) was related to known values of $\dot{q}_w$ in a calibration procedure described in Ref. 18.

Results

A comparison of orifice-cavity pressures P_0 and tube-orifice pressures P_c over the length of the model is shown in Fig. 2. In

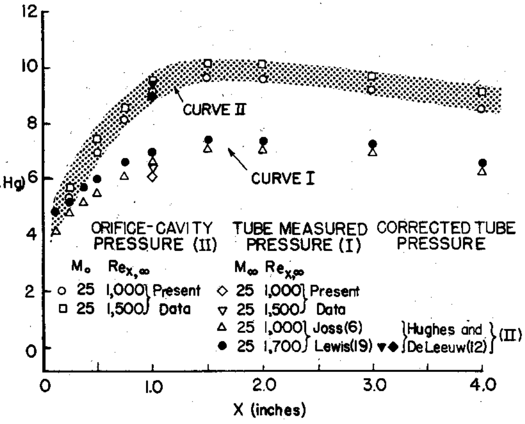

Fig. 2 Tube-Orifice pressure converted to orifice-cavity pressure.

addition to the values of P_c obtained in the present investigation, values of P_c obtained by Joss[6] and Lewis[19] under identical flow conditions are plotted in Fig. 2. (A small correction (1–2%) for thermal transpiration has been applied to these data.) The disparity between the two sets of data is convincing proof that tube-orifice pressures are not equivalent to orifice-cavity pressures under all flow conditions as was assumed in Ref. 1, i.e., the effects of S_i, L/d and α can be very important. Moreover, had the entrance diameter of the tube-orifices been the same as that for the orifice-cavities, this disparity would be slightly larger, assuming L remained the same. L is essentially a function of the model thickness so the latter is a reasonable assumption. The values of incident molecule speed ratio S_i vs distance from the leading edge were determined from the other measured quantities using the procedure described earlier.[18] Since the models used by Joss and Lewis were available, the values of L/d and α were determined for each tube-orifice. Using these values of S_i, L/d and α, the tube-orifice pressures in Fig. 2 were converted to orifice-cavity pressures using the theory of Hughes and DeLeeuw.[12] The converted values all fell within the shaded region in Fig. 2, i.e., they agreed with the measured values of orifice-cavity pressure, thus verifying the Hughes and DeLeeuw theory. This means that one can not assume that all molecules passing through the entrance plane of a tube orifice equilibrate in the transducer cavity and thus contribute to the measured cavity pressure under these flow conditions. Some of the molecules strike the tube wall and are reflected back out of the orifice without equilibrating. This is considered a direct evaluation of a key assumption made in the development of earlier correction schemes.[1,2]

The apparent agreement between the orifice-cavity pressures and the tube-orifice pressures near the leading edge is fortuitous; the tubes being slightly inclined toward the freestream (as in Fig. 1b) compensated for the effects of high-speed ratio and finite L/d, as evidenced by the small correction according to the Hughes and DeLeeuw theory.

We can also look at the values of P_w calculated from our orifice-

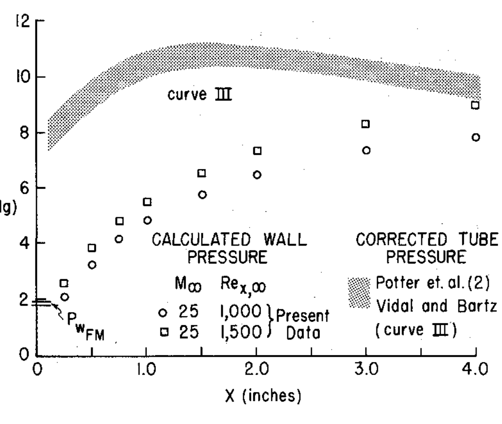

Fig. 3 Calculated wall pressure vs distance from the leading edge.

cavity pressure and the two stream Maxwellian model and compare these with the values of P_w obtained from tube-orifice pressures which have been corrected with the previously mentioned empirical correction schemes.[1,2] This comparison is shown in Fig. 3, and it shows the limitations of the empirical correction schemes. The values calculated from the orifice-cavity pressures using the two stream Maxwellian model are considered to be the correct values since they approach the free molecule value near the leading edge and approach the continuum strong interaction theory near the trailing edge.[18] This is the first time that wall pressures have been shown to approach the free molecule value at the leading edge although measurements of heat-transfer rate and shear stress have previously been shown to approach their respective free molecule values.[1] The values of P_w obtained from the tube-orifice measurements corrected according to the empirical schemes,[1,2] approach the orifice-cavity results at the furthest station downstream. At this position S_i has decreased to 3.5 from its freestream value of 20, and therefore the effects of S_i, L/d and α are as expected; quite small.

There has been some controversy about whether or not the wall pressure measured near the leading edge of a model which is long compared to a mean free path should even approach the free molecule value. The two principal factors which might lead one to believe that free molecular surface properties should not be observed for our test configuration are 1) Monte Carlo calculations of surface properties[20] for similar flow conditions which indicate that the wall pressure P_w at $x = 0.25$ in. in Fig. 3 should be approximately 10 μHg instead of 2 μHg, i.e., five times the free molecule value, and 2) measurements of number density[6] upstream of a sharp leading-edge flat plate which show that the number density is 50% or so larger than the undisturbed value due to the presence of plate emitted molecules which have traveled back upstream. In Ref. 18 one may find a good discussion of the spatial resolution required to accurately calculate or measure number density among other properties at the surface near the leading edge of a sharp flat plate. It is shown that these requirements have not been met in previous investigations[20,6] and thus erroneously high values of number density, and hence, molecular flux, surface pressure, shear stress, and heat transfer near the leading edge are reported. Physically, the leading edge cuts off half of the incident molecule distribution function, i.e., only those freestream molecules having a downward component of velocity can contribute to the number density at the surface just downstream of the leading edge. Thus surface number density should be approximately $0.5\, n_\infty$ whereas the calculations predict $1.3\, n_\infty$. The proposed scaling for this leading-edge "shadow effect" and the predicted value of number density at the surface were confirmed by the measurements presented in Ref. 18.

The influence of the observed upstream disturbance[6] on surface properties is more difficult to assess. However, a recent experimental and theoretical investigation[21] of the flow just upstream of the leading edge of a "razor blade" model, i.e., near zero thickness, indicates that the surface molecular flux should be 20%–40% greater than free molecule values just downstream of the leading edge for this flow condition. To this extent then, the other surface properties should approach their free molecular values.

Furthermore, we argue that the wall pressure should approach free molecule values since measured values of shear stress,[1] heat-transfer rate,[1,18] molecular flux[8] and number density[8] have been reported which approach their respective free molecule values under similar flow conditions. Indeed to argue that near free molecular wall pressures should not be observed, one would have to demonstrate that normal momentum is transferred in a much different fashion than either tangential momentum or kinetic energy, given the experimentally observed molecular flux[8] which approaches its free molecule value. Moreover, the model of the flowfield used here has been shown to be internally self consistent,[18] i.e., using the model and measured values the same value of U_i has been calculated two different ways. [Refer to the discussion following Eq. (8).]

Guidelines for Measuring Surface Pressures in Low-Density Hypersonic Flows

Assuming that flush mounted piezoelectric transducers cannot be used, one must use either sharp-edged orifice-cavities or tube-orifices. In regions of the flow where the flow adjacent to the surface has a high speed ratio (Mach number), the usefulness of tube-orifices is very much inferior to that of the orifice-cavities for the following reasons. 1) The continuum assumptions used in developing the analytic expressions for the tube-orifice corrections are not valid when large gradients in the tube exist over distances of the order of a mean free path. 2) When the speed ratio of the incident stream is high, the effects of finite L/d and angle of incidence between the tube-orifice and the surface it is mounted in are very large.[12] This leads to a lack of reproducibility from model to model since small changes in L/d and angle of incidence produce large changes in measured values, even before any corrections are applied. 3) On curved surfaces the fabrication of an orifice-cavity may be easier to assure a sharp edge entrance than it is to make a tube-orifice normal to the surface. We have had success using a piece of 0.001 in. shim stock with an orifice of the desired diameter brazed over a larger diameter hole drilled into the model. This ensures an $L/d \leqq \frac{1}{20}$. One can, however, use the pressures measured with tube-orifices in conjunction with measured values of S_i, L/d, and α to compute an equivalent orifice-cavity pressure using the theory of Hughes and DeLeeuw, provided that $\lambda > d$. (It would seem more straightforward to measure orifice-cavity pressures in the first place!) One can then determine the wall pressure P_w from these equivalent orifice-cavity pressures and a model of the gas-surface boundary and additional measurements.

In a situation where $d \gg \lambda$, the low density effects vanish and generally speaking, the slip velocities become small, and the tube-orifice technique is superior. There is a grey region though, for $0.1 < \lambda_0/d < 1.0$ in which neither technique may be accurate. In this case one can use a data reduction scheme proposed by Harbour and Bienkowski.[17] The principle of their approach is to use the data of Ref. 2 to convert the measured cavity pressure (the cavity pressure one would measure if $\lambda \gg d$). This value of cavity pressure is then used to deduce the surface molecular flux and wall pressure in a manner similar to that described above.

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